AP CHEMISTRY

TOPIC 10: NUCLEAR CHEMISTRY, PART B,

Day 118:

- Half-life (Nuclear)

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1. The half-life of Iodine-125 is 60 days. If the original sample had a mass of 50.0 grams, how much remains after 360 days?

$$(\text{mass})_t = (\text{mass})_0 \, (0.5)^n \quad OR \quad (\text{mass})_t = (\text{mass})_0 \, (\tfrac{1}{2})^n$$

$$n = \frac{360\ days}{60\ days} = 6$$

$$(\text{mass})_t = (50.0\ \text{grams})(0.5)^6 = 0.781\ \text{grams}$$

OR

$$\ln\left(\frac{[A]_t}{[A]_0}\right) = -kt \quad ; \quad \frac{\ln(0.5)}{-t_{1/2}} = k = \frac{\ln(0.5)}{-60\ days} = 0.011552\ \frac{1}{days}$$

$$\ln\left(\frac{[A]_t}{[A]_0}\right) = -kt \quad ; \quad \ln\left(\frac{[A]_t}{50\ g}\right) = -\left(0.011552\ \frac{1}{days}\right)(360\ days)$$

$$\ln\left(\frac{[A]_t}{50\ g}\right) = -4.158883083$$

$$e^{\ln\left(\frac{[A]_t}{50\ g}\right)} = e^{-4.158883083} \quad ; \quad \frac{[A]_t}{50\ g} = 0.015625$$

$$[A]_t = (0.015625)(50\ g) = 0.781\ g$$

2. Titanium-51 has a half-life of 6 minutes. If 98.0 grams of the material are obtained initially, how many grams of the sample will remain after one hour?

$$(\text{mass})_t = (\text{mass})_0 \times 0.5^n$$

$$n = \frac{60 \text{ min}}{6 \text{ min}} = 10$$

$$(\text{mass})_t = (98.0 \text{ grams})(0.5^{10}) = 0.0957 \text{ grams}$$

OR

$$\ln\left(\frac{[A]_t}{[A]_0}\right) = -kt \quad ; \quad \frac{\ln(0.5)}{-t_{1/2}} = k = \frac{\ln(0.5)}{-6 \text{ min}} = 0.11552 \frac{1}{\text{min}}$$

$$\ln\left(\frac{[A]_t}{[A]_0}\right) = -kt \quad ; \quad \ln\left(\frac{[A]_t}{98 \ g}\right) = -\left(0.11552 \frac{1}{\text{min}}\right)(60 \text{ min})$$

$$\ln\left(\frac{[A]_t}{98.0 \ g}\right) = -6.931472$$

$$e^{\ln\left(\frac{[A]_t}{98.0 \ g}\right)} = e^{-6.931472} \quad ; \quad \frac{[A]_t}{98.0 \ g} = 0.000976532$$

$$[A]_t = (0.000976532)(98.0 \ g) = 0.0957 \ g$$

3. The half-life of phosphorus-30 is 2.5 min. What percent of phosphorus-30 nuclides would remain after 14 min?

$$\ln\left(\frac{[A]_t}{[A]_0}\right) = -kt \quad ; \quad \frac{\ln(0.5)}{-t_{1/2}} = k = \frac{\ln(0.5)}{-2.5 \ \text{min}} = 0.27725887 \ \frac{1}{\text{min}}$$

Start with ANY amount – I like 100 grams

$$\ln\left(\frac{[A]_t}{[A]_0}\right) = -kt \quad ; \quad \ln\left(\frac{[A]_t}{100 \ g}\right) = -\left(0.27725887 \ \frac{1}{\text{min}}\right)(14 \ \text{min})$$

$$\ln\left(\frac{[A]_t}{100 \ g}\right) = -3.8816242$$

$$e^{\ln\left(\frac{[A]_t}{100 \ g}\right)} = e^{-3.8816242} = 0.020617311$$

$$\frac{[A]_t}{100 \ g} = 0.020617311 \quad ; \quad [A]_t = (0.020617311)(100 \ g) = 2.06 \ g$$

$$\frac{2.06 \ g}{100 \ g} \ x \ 100\% = 2.06\%$$

OR

$(\text{mass})_t = (\text{mass})_0 \text{ x } (0.5)^n$

All we are concerned with is the percent, so use only

$(0.5)^n$ portion of the equation:

$$n = \frac{14 \ \text{min}}{2.5 \ \text{min}} = 5.6$$

$$(0.5)^{5.6} = 0.020617311$$

percent is equal to: 0.020617311 x 100% = 2.06%

4a. If a 1.0 x 10^{-3} gram sample of technetium-99 has a decay rate 6.3 x 10^{5} nuclei sec^{-1}, what is its decay constant?

$$Rate = k\,N_t$$

$$\frac{1.0\times10^{-3}\,gram\ Tc}{}\times\frac{1\ mol\ Tc}{99\ \ grams}\times\frac{6.022\times10^{23}\,nuclei}{1\ mol\ Tc}\ =\ 6.08\times10^{18}\ \ {}^{99}_{43}Tc\ nuclei$$

$$k\ =\ \frac{Rate}{N_t}\ =\ \frac{6.3\times10^{5}\ \ nuclei\ \ \sec^{-1}}{6.08\times10^{18}\ \ nuclei}\ =\ 1.036\times10^{-13}\ \ \sec^{-1}$$

4b. What is the half-life of technetium-99?

$$k\ =\ 1.036\times10^{-13}\ \ \sec^{-1}$$

$$t_{1/2}\ =\ \frac{\ln(0.5)}{-k}\ =\ \frac{\ln(0.5)}{-1.036\times10^{-13}\ \ \sec^{-1}}\ =\ 6.69\times10^{12}\ \ \sec$$

5. How many half-lives have elapsed if a substance with an initial mass of 35.6 grams currently has a mass of 1.75 grams after 6.44 hours? (4.35 half-lives)

$$\ln\left(\frac{[A]_t}{[A]_0}\right)=-kt\quad ;\quad \frac{\ln\left(\frac{[A]_t}{[A]_0}\right)}{-t}\ =\ k\quad ;\quad \frac{\ln\left(\frac{1.75\ \ g}{35.6\ \ g}\right)}{-6.44\ \ hrs}\ =\ k\ =\ 0.4678152\ \ \frac{1}{hrs}$$

$$\frac{\ln(0.5)(hrs)}{-0.4678152}\ =\ t_{1/2}\ =\ 1.48166807\ \ hrs$$

$$n\ =\ \frac{6.44\ \ hrs}{1.48166807\ \ hrs}\ =\ 4.35\ =\ number\ \ of\ \ half-lives$$