AP CHEMISTRY

TOPIC 10: NUCLEAR CHEMISTRY, PART B, EXAMPLES Day 118:

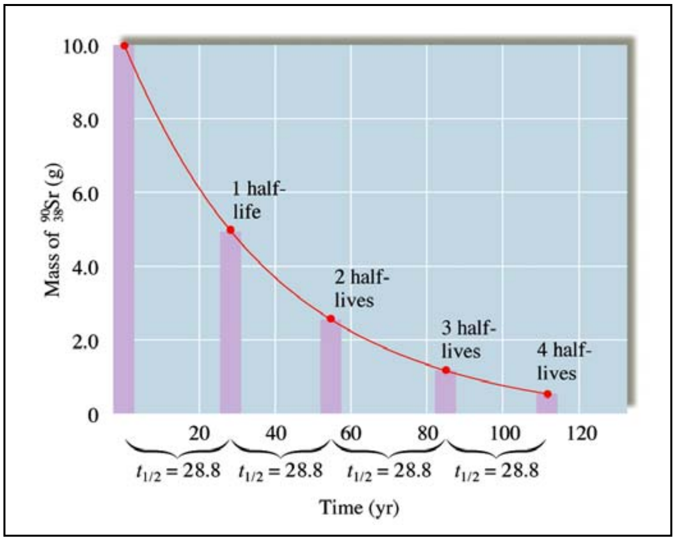
- Half-life (Nuclear)

HALF-LIFE:

A common way to describe the rate at which radioactive decay is occurring is a measurement known as **half-life**. Half-life is defined as the time necessary for one half of a radioactive sample to undergo decay into new elements.

Different isotopes have different decay rates. Some are as long or longer than 4.5 million years (Uranium-238) to as short as 10-microseconds (astatine-215, $^{215}_{85}At$). It is not necessary to understand the factors that contribute to the length of the half-life. There is also no way to predict when an atom will undergo a single decay, but it possible to observe large amounts of decay and come up with an AVERAGE rate.

If the amount of a radioactive substance is measured over time and the results are plotted, the resulting graph is known as a decay curve. The graph below shows the decay pattern more clearly. Also, these are all FIRST-ORDER decay rates.



From a conceptual approach, we can look at half-life as the time it takes for ½ of a sample to decay into some other substance. For instance, if we start out with 1.0 gram of a radioactive sample, after one half-life has elapsed, we will have 0.5 gram of the original material. After two half-lives, we will have 0.25 gram. After 3, 0.125 gram. As you can see, this could go on for some time, but it is generally accepted that after about 10 half-lives have elapsed, there is a negligible amount of the original radioactive material left.

Most of the problems on the AP exams that contain half-lives are relatively simple to solve, using either conceptual or mathematical approaches. On the exam, you are not provided with any equations related to nuclear chemistry. Therefore, any calculations you will have to make should be fairly simplistic and easy to solve using a few simple rules.

An equation that I gave I gave in general (honors) chemistry was the equation to calculate the mass (or amount) of material remaining after a certain number of half-lives.

$$(\text{mass})_t = (\text{mass})_0 \times 0.5^n$$

Where $(\text{mass})_t$ is equal the mass after a certain number of half-lives. $(\text{mass})_0$ is equal to the initial amount. "n" is equal to the number of half-lives.

Example #1:

After 150 days, what is the mass of a sample of a certain substance with an initial mass of 100 grams, with a half-life of 27 days? (answer: 2.13 grams)

$$(\text{mass})_t = (\text{mass})_0 \text{ x } 0.5^n$$

$$n \; = \; \frac{150 \;\; days}{27 \;\; days} \; = \; 5.556$$

$$(\text{mass})_t \; = \; (\text{ 100.0 grams }) \; (\; 0.5^{5.556} \;) \; = \; 2.13 \text{ grams}$$

OR

$$\ln\left(\frac{[A]_t}{[A]_0}\right) = -kt \qquad k \; = \; \frac{\ln(0.5)}{-t_{1/2}} \; = \; \frac{\ln(0.5)}{-27 \;\; days} \; = \; 0.0256721 \; \frac{1}{days}$$

Pick any mass (I like 100 grams)

$$\ln\left(\frac{[A]_t}{100 \;\; g}\right) \; = \; -\left(0.0256721 \; \frac{1}{days}\right)(150 \;\; days) \; = \; -3.8508177$$

$$e^{\left(\ln\left(\frac{[A]_t}{100 \;\; g}\right)\right)} \; = \; e^{-3.8508177}$$

$$\frac{[A]_t}{100 \;\; g} = \; 0.021262343$$

$$[A]_t \; = \; (0.021262343)(100 \;\; g) \; = \; 2.13 \;\; g$$

Example #2: (**AFTER** PART II explanation)

How many half-lives have elapsed if a substance with an initial mass of 100 grams currently has a mass of 33 grams after 40 minutes? (1.6 half-lives)

$$\ln\left(\frac{[A]_t}{[A]_0}\right) = -kt \quad ; \quad \ln\left(\frac{33\ g}{100\ g}\right) = -k(40\ \text{min})$$

$$\frac{\ln\left(\frac{33\ g}{100\ g}\right)}{-(40\ \text{min})} = k = 0.0277166\ \frac{1}{\text{min}}$$

$$\frac{\ln(0.5)(\text{min})}{-(0.0277166)} = t_{1/2} = 25.0\ \text{min}$$

$$n = \frac{40\ \text{min}}{25\ \text{min}} = 1.6$$

Example #3: (**AFTER** PART II explanation)

How much time has elapsed if a substance with an initial mass of 242 grams currently has a mass of 24.2 grams? The substance has a half-life of 5.25 days. (17.4 days)

$$\ln\left(\frac{[A]_t}{[A]_0}\right) = -kt \quad ; \quad \ln(.5) = -kt_{1/2} \quad ; \quad \frac{\ln(0.5)}{-t_{1/2}} = k$$

$$\frac{\ln(0.5)}{-5.25\ days} = k = 0.132028\ \frac{1}{days}$$

$$\ln\left(\frac{[A]_t}{[A]_0}\right) = -kt$$

$$\frac{\ln\left(\frac{[A]_t}{[A]_0}\right)}{-k} = t$$

$$\frac{\ln\left(\frac{24.2}{242}\right)(days)}{-0.132028} = 17.4\ days$$